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FIG. 8. The A' and D' parameters (in deg<sup>-1</sup>) for the logarithmic expression of the specific heat.

scale. It should be noted that these results extend an order of magnitude closer to the transition than previous data at saturated vapor pressure. This is due in part to the improved density resolution of the apparatus. However, as has been noted, pressure equilibrium times are more favorable at pressures above 1 atm so that our low-pressure data are not of as high quality as that shown in Fig. 6 and are of comparable resolution to the vapor-pressure work.<sup>2–4</sup>

We have analyzed our data in terms of a logarithmic singularity in temperature displacement from  $T_{\lambda}$ :

$$\begin{aligned} & u_P = A_{\rm I} \log_{10} | T - T_{\lambda} | + D_{\rm I} \quad T > T_{\lambda} \\ & u_P = A_{\rm II} \log_{10} | T - T_{\lambda} | + D_{\rm II} \quad T < T_{\lambda} \\ & \text{for } 10^{-2} > | T - T_{\lambda} | > 2 \cdot 10^{-5} ^{\circ} \text{K}. \end{aligned}$$
(6)

It is apparent from Fig. 6 that  $A_I \neq A_{II}$ . The various coefficients for these equations for the data at each pressure are plotted in Fig. 7 as a function of the density along the  $\lambda$  line. In all cases the fit to the data is good until  $|T - T_{\lambda}| = 10^{-2}$ °K. As can be seen, the slope parameters increase sharply at higher densities. On a plot of A versus pressure this increase is more linear. The low-pressure results are in reasonable agreement with the vapor-pressure results of Chase *et al.*<sup>3</sup> and Kerr and Taylor<sup>4</sup> with our uncertainty corresponding roughly to the difference between their values.

The specific heat  $C_P$  may be calculated from our data in a region close to the transition by means of the relation

$$\frac{C_P}{T} = \left(\frac{\partial S}{\partial T}\right)_{\iota} + \left(\frac{dP}{dT}\right)_{\lambda} \left(\frac{\partial V}{\partial T}\right)_{P},\tag{7}$$

where  $t = T - T_{\lambda}$  at constant pressure. We assume that over the range  $10^{-2} > |T - T_{\lambda}|$  the slope  $(\partial S/\partial T)_t = (dS/dT)_{\lambda}$ , and by using the entropy data of Lounasmaa and Kojo<sup>22</sup> along the  $\lambda$  line and the present results for  $(dP/dT)_{\lambda}$ ,  $V_{\lambda}$ , and  $\alpha_P$ , we have obtained the results shown in Fig. 8. The validity of this estimate of  $C_P$  extends of course only over the range of the  $\alpha_P$  data. We note that the change in slope is relatively much less than that for the  $\alpha_P$  coefficients. This is because the increase of the  $\alpha_P$  coefficients with pressure is compensated for by the corresponding decrease of  $(dP/dT)_{\lambda}$  and we note that the increase in the  $C_P$  parameters at low pressures is due to the strong increase of the slope of the  $\lambda$  line. Our calculations give results that extrapolate reasonably well to the experimental results of Kellers<sup>6</sup> (Fig. 8).

It is now of interest to check the consistency of our data with other recent results at elevated pressures. Our values of  $V_{\lambda}(T)$  and of the coefficients of  $\alpha_P$  given by the lines in Fig. 7 in effect completely define the volume surface near the  $\lambda$  line. Thus we may calculate (by numerical interpolation on a computer) V at any point between  $10^{-2}$  and  $10^{-5}$ °K separation the  $\lambda$  line and in particular may determine the shapes of the isochores and isotherms in this interval. Thus, while there are



FIG. 9. Parameters A''' and D''' (in atm deg<sup>-1</sup>) for a logarithmic fit to the slope  $(\partial P/\partial T)_V$ . Black circles and squares: present results. Open triangles: Lounasmaa and Kaunisto (Ref. 23); open circles and squares: Lounasmaa (Ref. 25), + Kierstead (Ref. 24).

<sup>22</sup> O. V. Lounasmaa and E. Kojo, Ann. Acad. Sci. Fennicae Ser. AVI, No. 36 (1959).

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